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The spatial distribution of spheres falling in a viscous liquid

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The spatial distribution of uniformly sized spheres falling in a viscous liquid is investigated experimentally for a solid volume fraction of 0·025 and a single sphere Reynolds number of 0·6. The observed spatial distribution agrees closely with a random distribution based on allocation of spheres to space cells according to a binomial probability mechanism.

1. Introduction

It is well established that the spacing of the individuals in an infinite system of free solid particles falling through a viscous fluid is non-uniform. Theoretical analyses for uniformly distributed systems, such as that by Happel (1958), give settling velocities substantially different from observed velocities. More direct evidence is provided by the observations of Kaye & Boardman (1962) and of Johne (1966) of the fluctuations in the velocities of individuals within a system of particles settling from suspension.

The simplest irregular distribution which can be suggested plausibly for a system of settling particles is that where the particles are allocated to positions in space at random. Such a distribution is the basis for the computations of settling velocities by Burgers (1942) and by Famularo & Happel (1965) and for the formulation of parameters in the correlation by Johne (1966).

While the assumption of a random spatial distribution is a natural one for a system of freely distributed particles, it has been applied to falling particles without substantial justification. There are, indeed, grounds for the belief that some order should be established in a system of falling particles by hydrodynamic interactions between the individuals. Evidence for such interactions can be found in the observations of Jayaweera, Mason & Slack (1964) and in the computations of Hocking (1964) which describe motions within a small group of falling particles. The sort of order envisaged for an infinite system of particles is not any regular order but an equilibrium state of partial order with the continual formation and dispersion of sympathetic groups of particles.

The investigation reported in this paper is an experimental determination of the spatial distribution of freely falling solid spheres and a comparison of this distribution with a random distribution.

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2. Experimental

In order to facilitate observation and to avoid the possibility of substantial interparticle forces due to surface effects, large particles, 4 mm acrylic spheres, were used for the experimental work. Silicone liquid of viscosity 1·00 poise was chosen as the settling medium. The Reynolds number of a single sphere settling at its Stokes terminal velocity in the liquid was 0·6, so that flow was in the viscous-dominated range. Tests were made in a 110 mm × 110 mm square settling vessel which was equipped with a perforated floor to permit suspension of the particles by fluidization.

Fluidization was the most satisfactory of the various techniques for dispersing the spheres in the fluid to the desired concentration. With agitation by a stirrer or by movement of the vessel containing the mixture, there are always doubts about residual motions and density currents. Fluidization, however, is relative particle-fluid motion of the same kind as in settling itself and, since Richardson & Zaki (1954) have shown that the relative velocities between particles and fluid are the same in the fluidization and settling processes, there is no reason to suppose that the particle configurations should differ.

During fluidization, slow circulation of spheres up the vessel axis and down its walls was observed, while during settling in still liquid the reverse circulation resulting in faster settling in the middle of the vessel occurred. Because of the danger of some interference with normal particle settling configurations, this 'wall effect' was to be avoided. It was found that when the particles were allowed to settle against a fluid velocity of about 0·8 of the fluidization velocity, circulation was not evident. Of course, it could not be expected that the vessel boundary effects would be neutralized by this technique. Subsequent analysis of the spatial distribution of the spheres, however, showed no variation of results with location within the sample volume.

The settling spheres, a mixture of eight different colours, were filmed from two adjacent sides of the settling vessel simultaneously. By examination of the two views on any frame of the film, it was possible, with due perseverance, to find the spatial location of each sphere.

Only one solid volume concentration, 0·025, was investigated. It became evident in the analysis of the films that the determination of spatial locations at greater particle concentrations would be very difficult and uncertain. Smaller concentrations were not examined because of the impracticability of fluidization as a mixing technique at lower solid fractions.

3. Random spatial distribution

Prerequisite to the assessment of the degree of order in the system of settling particles is the definition of an appropriate random spatial distribution as the basis for comparison.

What seems to be the proper distribution for free particles could be generated by allocating, within a fixed space volume, the requisite number of particle centres successively to available space co-ordinates at random.

If such a process were executed not with the number of spheres required to give the actual concentration of the system but with an unlimited number of spheres, a maximum concentration, such that no space large enough to admit a further sphere remained, would eventually be reached. A volume of size V could, at this maximum concentration C_M, accommodate a number N of uniform spheres defined by

$$N = VC_M/\tfrac{1}{6}\pi D^3,$$

where D is the sphere diameter. At any concentration C, the probability that any one of the N particle sites in the volume V is occupied is P, defined by

$$P = C/C_M.$$

The distribution of particles to space volumes of size V by this random process is, then, described by the Binomial frequency distribution,

$$f_n = \binom{N}{n} P^n(1-P)^{N-n},$$

where f_n is the frequency of volumes containing n particles.

To use this distribution function, it is, of course, necessary to have a value for C_M. In principle, this presents no difficulty. The generation process involves no special particle-contact rules such as does a close-packed allocation model. In practice, an exact evaluation of C_M seems hardly feasible so that it would be necessary to employ a 'Monte Carlo' computation technique. Fortunately, at the low-volume concentrations of interest in this study, the distribution function is not critically sensitive to small errors in C_M so that a refined guess at the value is adequate. According to the experiments of Scott (1960), a close-packed random bed of spheres has a concentration of about 0·62. This value could not be reached by the simple space allocation process since particles are not necessarily placed in contact. A simple cubic lattice just widely enough spaced to admit interstitial particles has a concentration of $\frac{\pi\sqrt{3}}{16}$ or 0·34. Such a structure would represent an unattainably low limit to the maximum concentration. The value chosen for C_M is 0·5.

4. Determination of the spatial distribution

An $8 \times 8 \times 8$ cm cubic volume in the middle of the $11 \times 11 \times 40$ cm volume of falling particles was chosen as the sample space. At the volume concentration 0·025, the mean number of particles in this space was 384. An instantaneous number distribution of the particle centres to the 512 component 1 cm cubes of the sample space was determined in each of four independent settling tests. Each of the four settling tests gave very similar distributions which could not be distinguished from one another by standard statistical methods. The four results were treated, therefore, as a combined sample of 2048 1 cm cells.

In analysing the films it was clear that errors of identification of particles could occur. Such errors should not, however, produce definite bias of the distribution and would be few in number. Few in number, likewise, were particles for which the location could not be decided. These particles, up to 10 in number in the sample volume, were subsequently allotted to the 1 cm cells at random. The type of error introduced by this procedure would be a tendency to make the distribution more even because the missing spheres are more likely to have been hidden in denser parts of the sample space. The magnitude of this error is considered to be negligibly small.

5. Comparison of the observed and random distributions

At the concentration 0·025, the average number of 4 mm spheres in the 1 cm cube is 0·75. The number distribution in 1 cm cubic cells should, therefore, be decisive for the detection of any close-ordering in the system. The observed and random distributions in the 2048 cells are given in table 1.

No. of spheres in cell	No. of cells	
	Random	Measured
0	949	925
1	749	787
2	276	271
3	62·9	57
4 and more	11·1	8

TABLE 1. Distribution of spheres in 1 cm cells

No. of spheres in cell	No. of cells	
	Random	Measured
0	220	217
1	347	330
2	265	293
3	130	136
4	46	33
5 and more	16	15

TABLE 2. Distribution of spheres in $1 \times 1 \times 2$ cm cells

There is close correspondence between the observed and the random spatial distributions. The degree of correspondence can, perhaps, be appreciated fully when it is realised that in a system of spheres distributed uniformly in space there would be 512 cells containing no sphere and 1536 cells containing one sphere.

Indeed, it cannot be asserted that the deviation of the measured distribution from the random distribution in 1 cm cells indicates ordering at all, however slight, since an application of the Chi-Square statistical test to the two distributions gives a probability of 0·40 that a deviation of this size might arise in sampling from the random distribution.

Short-scale vertical ordering can be investigated by a comparison of the number distributions in vertical blocks formed by combining the 1 cm cubic cells. The figures for $1 \times 1 \times 2$ cm blocks are given in table 2.

Again, the deviation of the measured distribution from the random distribution is small. According to the Chi-Square test, a deviation as large as this would occur with a frequency of 0·17 in sampling from the random distribution. While it can be argued that this result indicates the existence of some vertical ordering in the settling spheres, it must be conceded that the degree of such ordering is slight.

A comparison of the number distributions in $2 \times 2 \times 1$ cm horizontal blocks, to test for horizontal ordering, shows a small deviation with a probability of 0·75 that it could have occurred in sampling from the random distribution.

For larger combined blocks, representing larger scales of possible ordering, deviations from the random distribution are again small and within the likely bounds of sampling variations. In $2 \times 2 \times 2$ cm blocks, the probability of a deviation as large as that measured is 0·45. For $2 \times 2 \times 4$ cm and $4 \times 4 \times 4$ cm blocks, the probabilities are 0·83 and 0·85 respectively.

6. Conclusion

The experimental results indicate that there is little spatial ordering of spheres falling in a viscous liquid. There is, in the results for the $1 \times 1 \times 2$ cm blocks, a suggestion of some short scale vertical ordering but the degree of order indicated is slight.

Evidently, the random spatial distribution is a substantial basis for a theoretical analysis of the settling of uniform particles. While the experimental results confirm this only for the volume concentration 0·025, there is no reason to suppose that the spatial distribution is not nearly random at smaller concentrations where sympathetic motions of the particles are, presumably, less strong. At larger concentrations, diminished particle separation makes the difference between a random and an ordered spatial distribution less significant in the computation of settling velocity.

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